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Experimental Simulation of Hypervelocity Flight

Referent: Prof. Dr. R. J. Stalker

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EXPERIMENTAL SIMULATION OF HYPERVELOCITY FLIGHT its Relevance in Communication with, and Detection of, Re-Entry Vehicles

Prof. Dr. R. J. Stalker*

Summary

The electrical and radiative characteristics of the flow field surrounding a hypervelocity vehicle are determined by the temperature and the density of the free electrons in the gas. For simulating these characteristics, it is necessary to correctly simulate the chemical reactions occurring in the gas. This requires high flow densities and energies, implying that experiments must take place in pulsed facilities. Such a facility is the shock tunnel. When used with a free piston driver, and operated as a reflected shock tunnel, it is possible to achieve the conditions to simulate chemical reactions at speeds near Earth orbital values. For higher speeds, radiation losses provide a barrier to further development of reflected shock tunnels and, therefore, it appears likely that these will be limited to maximum flight speeds between 10 and 15 km/sec. It is possible that non-reflected shock tunnels may allow operation at higher energies, provided that special techniques are employed to ensure rapid starting of the nozzle flow. Such a shock tunnel is being developed at present.

* On study leave from the Australian National University, Canberra

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1. Introduction

The need for experimental simulation of hypervelocity flight presented itself relatively suddenly to experimental aerodynamicists, during the late 1950's. Up to that time, wind tunnels had been developed mainly through a series of incremental improvements, responding to a steady increase in the flight speeds of aircraft. The maximum speeds of interest then were approximately 1km/sec.

The advent of the long range ballistic missile and the beginnings of space exploration which followed, introduced a demand for aerodynamic data at speeds near Earth orbital values (7.9 km/sec). Experimental aerodynamicists responded by inventing and evaluating a large range of experimental facilities (1). These include: "Hypervelocity Ranges" in which models of flight vehicles were projected at high speeds through stationary air, "Gun Tunnels", in which the test gas was heated and compressed by a high speed piston, before expansion through a hypersonic nozzle, "Hotshots", in which the test gas was heated by an electric arc, "Expansion Tubes", in which an expansion wave was used to accelerate and expand the test gas, "Shock Tunnels", which will be described later in this paper, and other devices. Many of the industrial test facilities which were built to exploit these various approaches are no longer operating, due to the contraction of research funding in this area. However, evaluation proceeded to the point where it was realized that different types of facilities were suitable for different special purposes, and in consequence, a number of the types which are most economical to operate, such as gun tunnels and shock tunnels, will be found in university.

The hypervelocities research group at the Australian National University is located in the Physics Department, and is interested in the high temperature physical phenomena occurring in hypervelocity flow fields. Therefore, a substantial effort has necessarily been devoted to extending the range of velocities beyond those at which other experimenters have worked, in order that these effects may readily be observed and studied. This paper discusses the methods used, the progress made, and some problems encountered in pursuing this objective.

2. Simulation Requirements

The experimental simulation of hypervelocity flight does not, of course, involve an attempt to reproduce, on a full scale flight vehicle, the conditions prevailing in flight. Rather, the purpose is to predict the conditions which will prevail in flight, by making investigations on small scale models of the full scale vehicle. In doing so, it is necessary to take into account the values of those parameters which govern the similarity between the flow field about the model and that about the full scale vehicle.

The most fundamental of these parameters is the Mach number. In flight, this is defined as the ratio between the flight speed and the speed of sound in the undisturbed air ahead of the vehicle. For a model in a wind tunnel, the corresponding quantities are the speed at which the tunnel test gas approaches the model, and the speed of sound in the gas immediately upstream of the model. The significance of the Mach number can be illustrated by reference to Fig. 1.

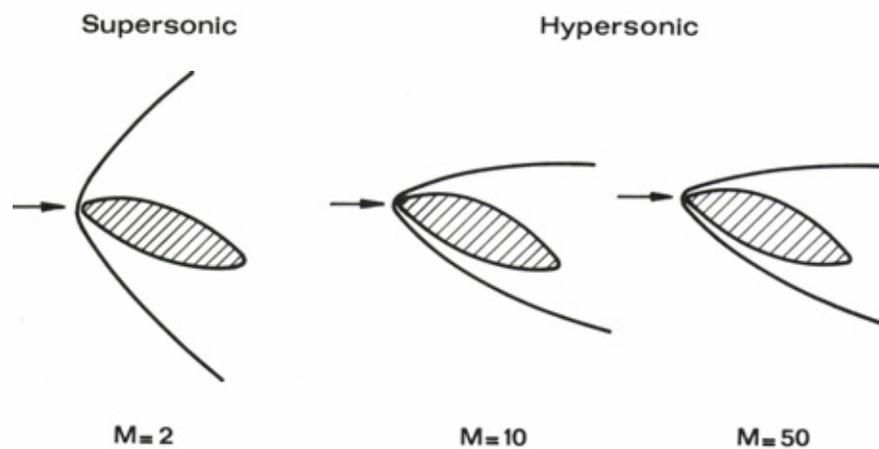


Fig. 1 Effect of Mach Number

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Here the flow pattern about a simple re-entry vehicle at various Mach numbers is represented. It can be seen that a change from a supersonic Mach number ($M=2$) to a hypersonic value ($M=10$) produces a strong effect, with the shock wave moving closer to the body, confining the flow field within a more limited zone close to the body. However, in passing from one hypersonic Mach number to another ($M=10$ to $M=50$) only slight changes take place and, for many practical purposes, these changes can be ignored. This "Mach Number Independence Principle" (2), is of particular importance in experimental simulation of hypervelocity flight, as it implies that many experiments may satisfactorily be performed at Mach numbers between 5 and 10, a factor which, in practice, considerably simplifies the design of experimental facilities.

The requirements for a suitable hypersonic value of the Mach number is common to all hypervelocity simulation experiments. Depending upon the particular hypervelocity phenomena which are to be investigated, there also may be requirements involving other simulation parameters. For example, to investigate boundary layer flow behavior, it would be necessary to pay attention to the Reynolds' number (involving the ratio of the inertia effects to viscous effects in the gas) as well as the relation between gas kinetic energy and body surface temperature. If the flow involved ablation of material from the body surface, then additional parameters, involving the properties of the material, would be involved. Obviously, in some cases, the full set of requirements for simulation may be very complicated and demanding.

The research at the Australian National University has tended to focus on simulation of the effects of chemical reactions in hypervelocity flow fields. One such effect, which is of particular interest here, is the production of free electrons. When these are present in the flow field, the gas becomes electrically conductive, and therefore the vehicle is surrounded by a conducting medium, which severely inhibits signal transmission to and from the vehicle, and increases its radar cross-section. Also, the free electrons tend to persist as the gas flows into the wake of the vehicle, and hence it leaves a trail of conducting gas as it passes through the atmosphere. The length of the trail depends on the size of the vehicle, and this fact is the basis of a method of discriminating between a ballistic missile, and its decoys, as they enter the atmosphere.

Up to a speed of approximately 7 km/sec, the predominant mechanisms for production of electrons involve some preliminary chemical reactions. This is illustrated in Fig. 2, where the preliminary reactions are the dissociation of oxygen, and the reaction of molecular nitrogen with atomic oxygen to produce nitrogen atoms. Oxygen atoms and nitrogen atoms then combine to produce ionized nitrous oxide, together with a free electron. It should be noted that Fig. 2 is an oversimplified version of what actually happens in the air, and that other reactions participate in producing nitrogen and oxygen atoms. However, it serves to demonstrate the important role which chemical reactions play in electron production.

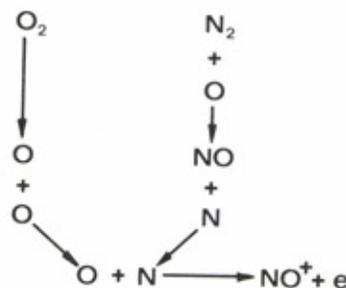


Fig. 2 Some Reactions Producing Electrons in High Temperature Air

In seeking to simulate the chemical reactions occurring in a hypervelocity flow field, we note that they follow the severe heating of gas, which occurs as it passes through the bow shock wave produced by the vehicle (see Fig. 3). Therefore, it is important to reproduce this heating, by ensuring that the gas kinetic energy associated with the velocity of the ambient atmosphere with respect to the flight vehicle is matched by the energy of the flow approaching the model in the wind tunnel. In addition, the reactions proceed at a rate which is determined by the speed at which the thermal energy of the gas is transferred into internal energy of excitation of the chemical bonds of the unreacted molecules, and this depends upon the frequency of collision between the gas particles. Thus in simulating the chemical reactions, it is necessary to arrange that an appropriate number of collisions between particles of the gas will take place as it passes through the vehicle flow field.

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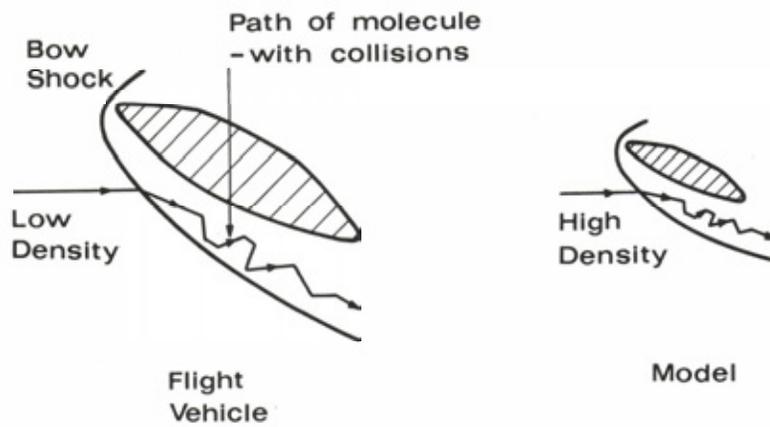


Fig. 3 Density Scaling for Chemical Reactions

As illustrated in Fig. 3, this implies that, in the wind tunnel, the smaller scale of the model, and the associated flow field, must be compensated by an increase in the density. For the type of reactions indicated in Fig. 2, where only two reacting particles are needed to produce a reaction, the reactions are simulated if the product of the density and a typical scale length (such as model length) are the same in the wind tunnel as in flight.

The consequences of these requirements for matching of stream energy, and of the product of density with model length, may be seen by reference to Fig. 4. The two variables determining the aerodynamics of a particular vehicle, namely its speed and its altitude, are taken as the axes on the figure, and the range of conditions over which flight is feasible is shown as the cross-hatched area. This area is usually referred to as the "flight corridor". To one side of the figure is a scale showing the kinetic energy per air molecule required to produce the speeds in the flight corridor. Remembering that one electron volt corresponds to a gas temperature of 10^4 °K, a consequence of the first requirement becomes obvious. Namely, that insoluble heating problems would be associated with attempts to maintain wind tunnel components in contact with air flow at such high energies for periods longer than a fraction of a second.

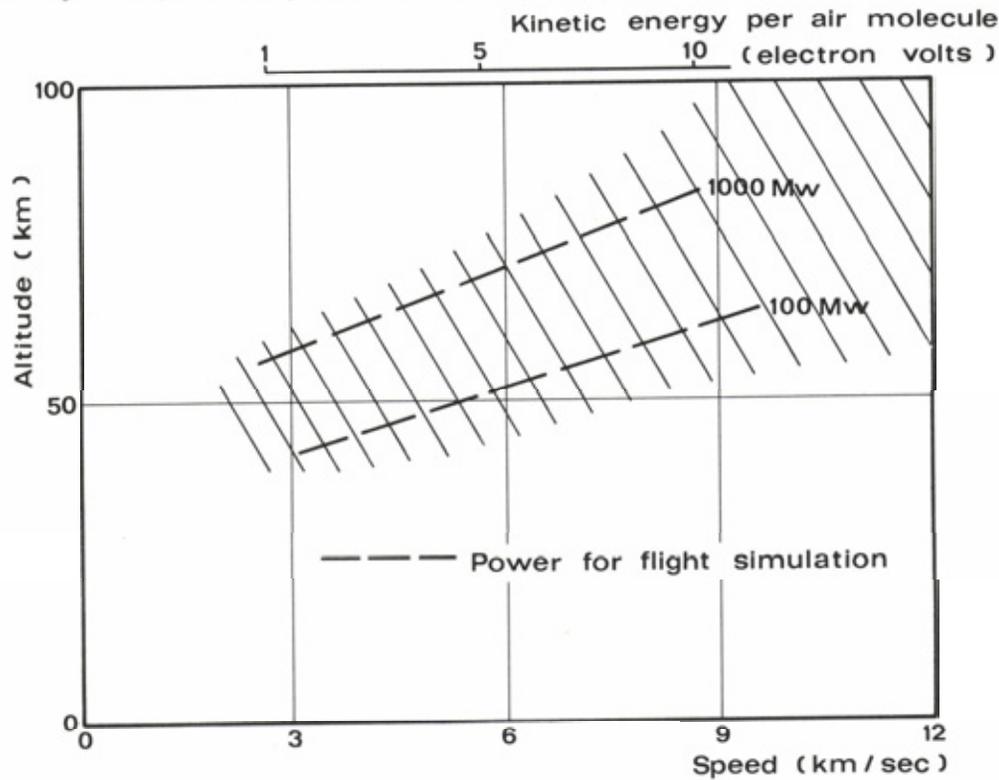


Fig. 4 Power and Energy Levels for Simulation with Chemical Reactions

Test Section Diameter: 30 cm
 Flight Vehicle Length : Test Model Length = 100:1

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The second requirement, when associated with the necessarily small scale of the model, implies that the densities in the wind tunnel are much higher than in flight and this, combined with the high velocities prevailing, leads to a very high energy flux through the test section of the tunnel. An indication of the power levels associated with this energy flux is given in the figure. As an example, a wind tunnel with a test section of 30 cm diameter has been chosen, together with a scaling factor of 100 between the flight vehicle size and that of the model. It can be seen that, for simulation within the flight corridor, power levels between 100 and 1000 MW are required. This is well beyond the capacity of conventional laboratory power sources.

Therefore, it is necessary to design the wind tunnel to operate on a single pulse basis, yielding test durations of the order of a few milliseconds, at most, and relying upon automatic techniques for recording experimental data. In this way, heating problems are avoided, and methods which allow high transient power can be used to drive the tunnel. For reaching high velocities, the shock tube is the most suitable of such methods (3). It can be coupled to a wind tunnel nozzle, in order to generate a hypervelocity flow, and the combination then is referred to as a "shock tunnel".

3. Shock Tunnel Operation

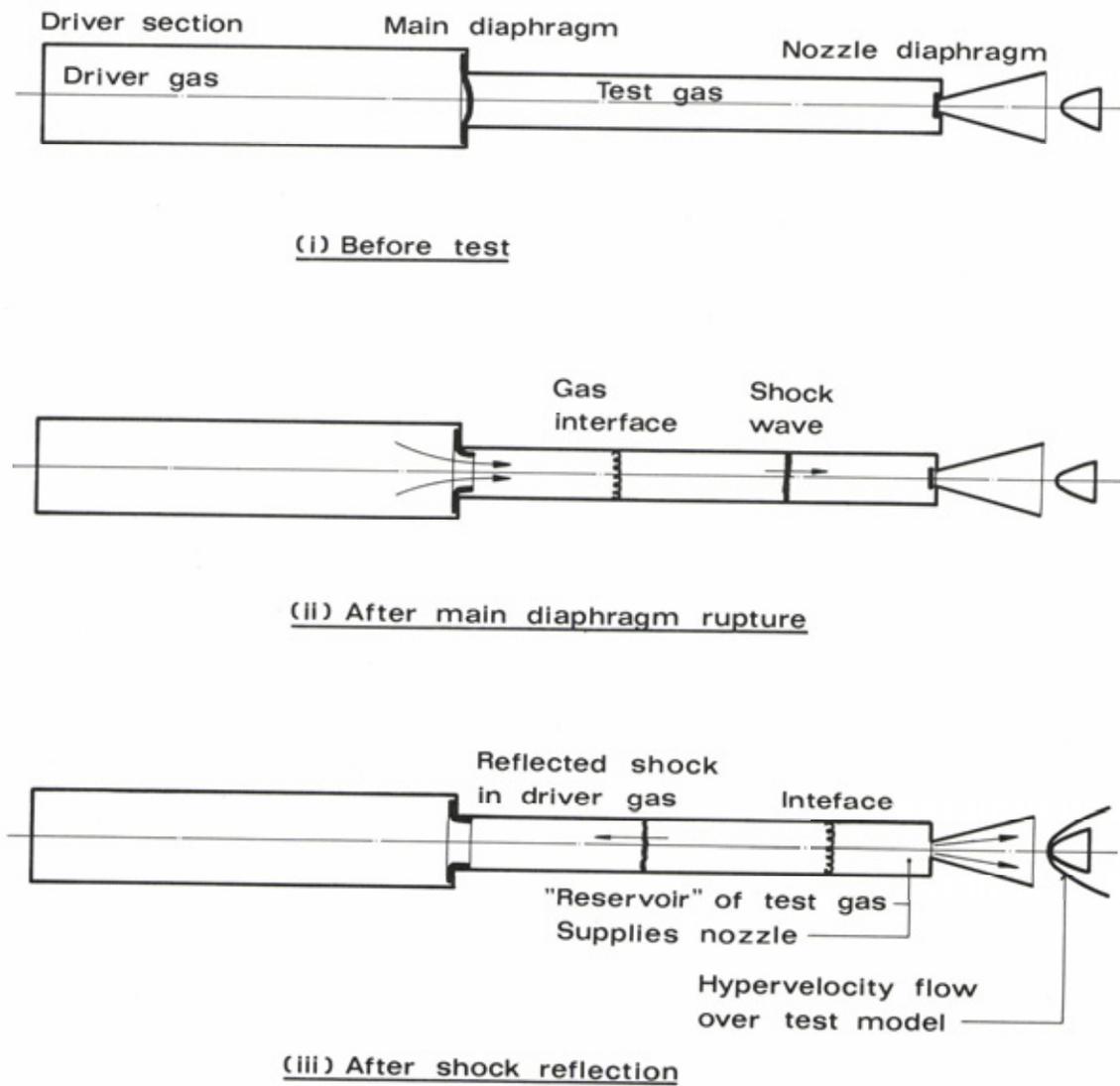


Fig. 5 Shock Tunnel Operation

The essential elements of a shock tunnel are displayed schematically in Fig. 5 (i). The shock tube contains the test gas and at one end, is separated from the driver section by a strong diaphragm. The driver section is filled with driver gas at high pressure. At the other end of the shock tube is a hypersonic nozzle, and the model is mounted at the nozzle outlet.

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Satisfactory pulsed operation of the hypersonic nozzle is only possible if the nozzle is evacuated before each test. To allow this, the nozzle is separated from the shock tube by a thin diaphragm.

Operation of the shock tunnel is initiated by rupture of the diaphragm between the shock tube and the driver. As shown in Fig. 5 (ii), expansion of the high pressure driver gas then drives the interface between the driver gas and the test gas along the shock tube. The interface is preceded by a strong shock wave in the test gas. When the shock wave reaches the end of the shock tube, it is reflected back towards the interface, leaving the test gas, which has now been compressed and heated by double passage of the shock wave, confined in a small volume at the end of the shock tube. At the moment of shock reflection, the thin diaphragm ruptures, allowing the test gas to begin flowing through the nozzle, and thereby to establish hypervelocity flow over the model.

The flow velocities obtained depend upon the energy transferred to the test gas from the driver gas. The transfer is facilitated by low density in the driver gas and, for this reason, high performance shock tunnels employ helium or hydrogen as driver gas, heated to as high a temperature as possible.

4. The Free Piston Shock Tunnel

In keeping with the pulsed nature of shock tunnels, the high temperature and pressure required in the driver gas also may be produced by a pulsed technique. This fact has been widely exploited in the past, and shock tubes with driver gas heated by an arc discharge (4,5) or by internal combustion of hydrogen and oxygen (6), were developed some years ago. As an alternative to these methods, and as a means for achieving higher energies, the free piston technique, for compressing and heating the driver gas, has been developed at the Australian National University (7).

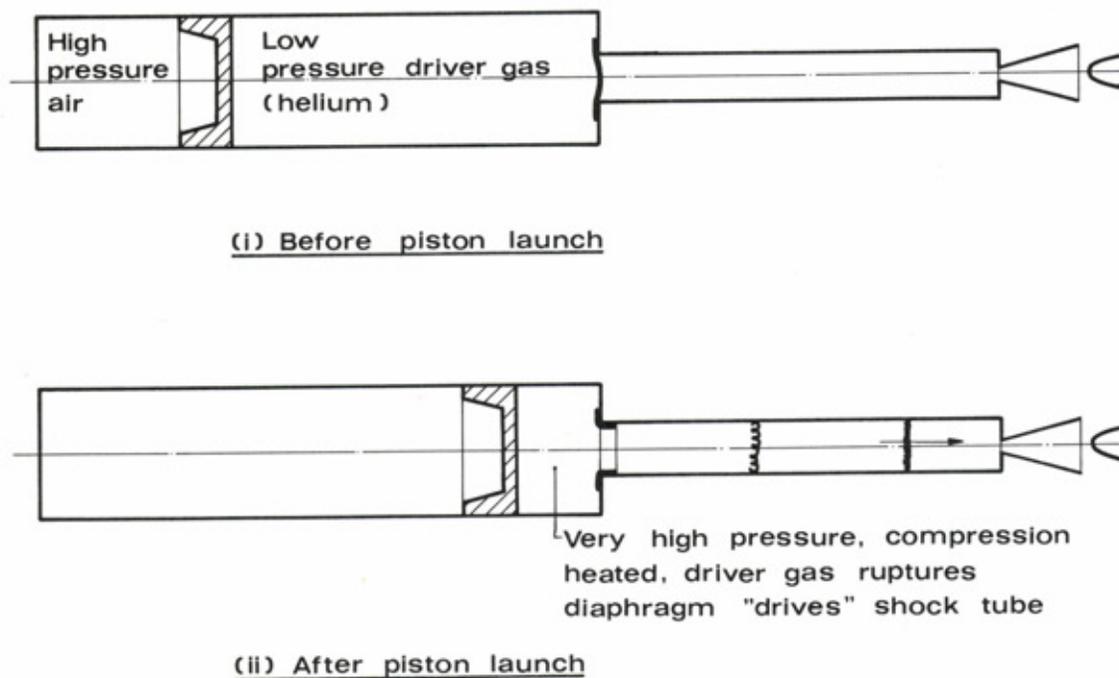


Fig. 6 Shock Tunnel with Free Piston Driver

This is illustrated in Fig. 6. The driver section consists of a relatively large tube, which is filled with driver gas at low pressure, and is separated from a reservoir of air at moderately high pressure by a free piston. Before a test, the piston is restrained in the position shown in Fig. 6 (i). The test is initiated by release of the piston, which is then projected along the tube, compressing and heating the driver gas as it goes. Upon completion of this compression stroke, the piston momentarily comes to rest, as shown in Fig. 6 (ii). At this point, all the energy obtained from the expansion of the reservoir gas has been delivered to the driver gas and, by arranging that the main diaphragm then ruptures, this energy is released to drive the shock tunnel.

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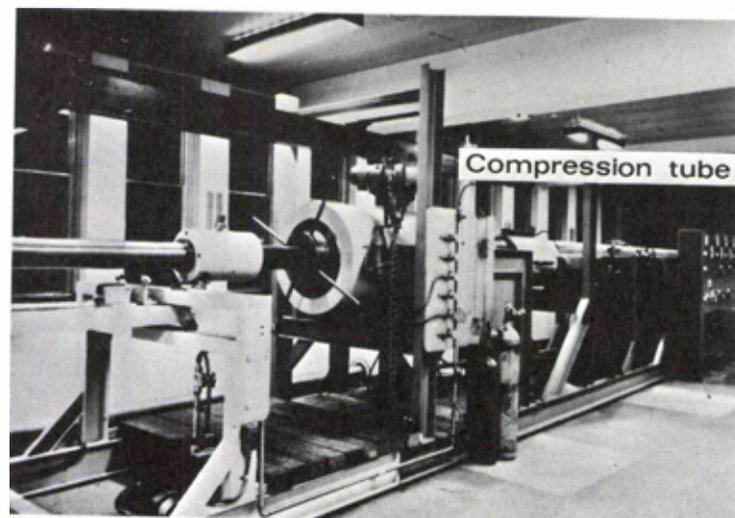
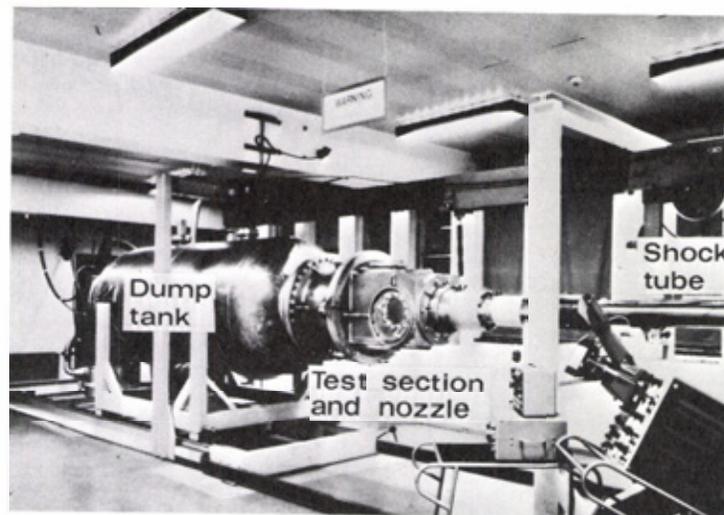


Fig. 7 Australian National University Free Piston Shock Tunnel

A composite photograph of the major free piston shock tunnel at the Australian National University is shown in Fig. 7 *. The driver gas compression tube has an overall length of 10 meters, of which 3.4 meters is taken up by the high pressure air reservoir. The piston travels over the remaining 6.6 meters, which is bored and honed to an internal diameter of 30 centimeters. The high pressure end of the compression tube, in the vicinity of the main diaphragm, is reinforced to withstand an internal pressure of 2700 atmospheres. The shock tube is 6.7 meters in length, and has an internal diameter of 7.6 centimeters. It is followed by a hypersonic nozzle, which has an exit diameter of 30 centimeters, and vents into a dump tank with a volume of 3.4 cubic meters. The test section is located at the nozzle exit, and windows are provided in the sidewalls at this point, in order that optical techniques may be used to study the flow about models.

The energy levels which may be achieved in this tunnel are well in excess of that required to study the aerodynamics of atmosphere re-entry at speeds corresponding to orbit about the Earth. At present routine operation at 10 electron volts/molecule is possible, with a test section power level of 60 MW. This is sufficient to allow experiments appropriate to the higher altitude portion of the flight corridor. Test durations are generally limited to one millisecond or less.

* Construction of this facility was financed by the Australian Research Grants Committee.

5. Experimental Observations - Flow Visualisation

The type of test data required from the flow about the models in the test section, depends on the particular experiments performed. For aerodynamic purposes, surface pressures and surface heat transfer rates often are required, and standard techniques now are available for making such measurements.

However, the effects of chemical reactions in the hypervelocity flows which are produced are most readily observed using optical techniques of flow visualisation. Examples of this are shown in Figs. 8 (a) and 8 (b). Here a Mach-Zehnder interferometer has been used to record details of the flow patterns over a cylinder (representing the blunt nose, or the leading edge of the flight vehicle) and an inclined flat plate (representing a rudimentary wing). The Mach-Zehnder interferometer responds to changes in the gas density, registering these as a series of light and dark contours of constant density, as in Fig. 8 (a), or as local shifts in a pattern of parallel fringes, as in Fig. 8 (b). In Fig. 8 (a), the pressure in the region between the shock wave and the surface of the cylinder is constant, and the series of density contours in that region indicates that the density is rising from the shock wave to the body, due to the rapid changes in temperature brought about by the chemical reactions in the gas. In Fig. 8 (b), interest is centered on the shock wave originating at the leading edge of the plate. In the absence of chemical reactions, the shock wave would be straight. However, the reactions induce shock curvature, and this can readily be seen by holding the paper to sight along the shock wave.

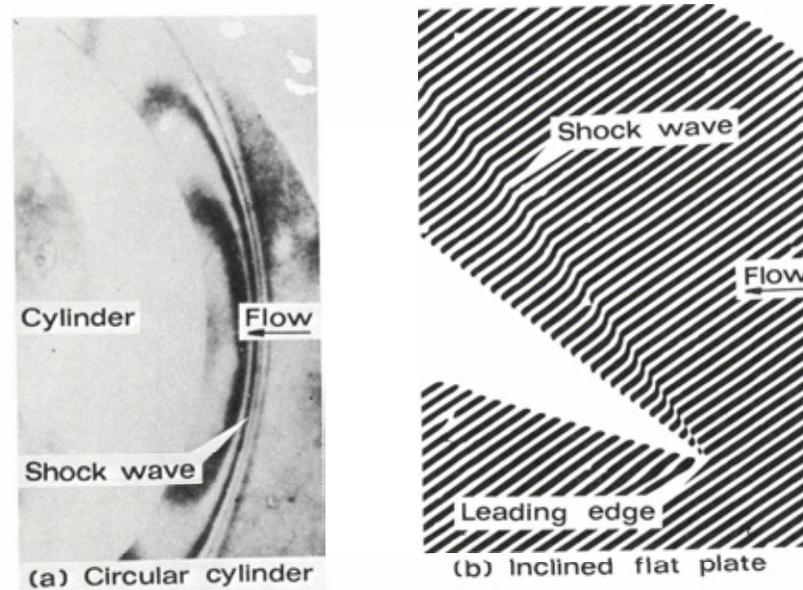


Fig. 8 Flow Visualisation using Mach-Zehnder Interferometer

There is a change in the surface pressure distribution associated with this curvature, and therefore a shift in the center of pressure on the plate.

The photographs presented in Fig. 8 have been obtained with carbon dioxide as the test gas, as the effects observed are most prominent. However, they also may be readily observed and studied in air, and such photographs provide the most powerful means we have at present for investigating the effects of chemical reactions in hypervelocity flow fields.

6. Radiation Losses in the Reflected Shock Tunnel

In principle, the velocities which may be obtained in a reflected shock tunnel are limited only by the maximum temperature in the driver gas, and it would appear that, provided this temperature can be increased, then there is no limit to the velocities which may be achieved in such a shock tunnel. In fact, radiation losses from the test gas, after reflection of the shock wave at the end of the shock tube, are responsible for a barrier to unlimited performance improvements.

The radiation is due to the high temperatures achieved in the gas in the reflected shock region. As these rise to values in excess of 15'000 °K, the thermal excitation of the electrons in the atoms of the gas leads not only to strong radiation from the atoms themselves, but also to an increasing

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amount of thermal ionization of the gas. The electrons so produced interact vigorously with the ionized atoms, to further increase the amount of radiation. The gas radiates at all frequencies but, because of the temperature involved, the strongest radiation occurs in the ultraviolet region.

To calculate the radiation power levels would be a simple matter if the gas were transparent. However, the temperatures and electron densities involved (values exceeding 10^{25} electrons m^{-3}) are such that the mean free path for radiation in the gas is much less than one millimeter and, as the dimensions of the shock tube are of the order of a few centimeters, the gas is very opaque. The radiation therefore escapes from the gas by a diffusion process, being absorbed and re-emitted many times by the electrons and atoms of the gas as it passes from the interior to the walls of the tube. Calculation of the radiation loss therefore becomes a relatively tedious and complicated process.

In order to develop a practical understanding of the effects of radiation loss, a shock tube study of these effects, with argon as the test gas, was made (8). Argon was chosen because it is a monatomic gas and, because there are no molecules with chemical bonds which may absorb energy (as in air), it will produce high levels of thermal ionization, and high radiation losses, at relatively modest shock velocities. Results of calculations of the radiative energy loss are shown in Fig. 9. Curves are shown for a number of shock tube diameters, showing that the rate of radiation loss is reduced as the size of the shock tube increases. In spite of this, the rate of energy loss is high, amounting to power levels of 1000 MW per liter of gas. Now, in reflected shock tunnel operation, the need to establish steady flow conditions over the model in the test section implies that there is an unavoidable lapse of time, after reflection of the shock wave at the end of the shock tube, before the shock heated gas is used. A large fraction of the initial energy is lost by radiation during this period and in fact, calculations for practical cases indicate that the energy remaining in the gas at the end of this period will never be sufficient to allow test section velocities much in excess of 8 km/sec.

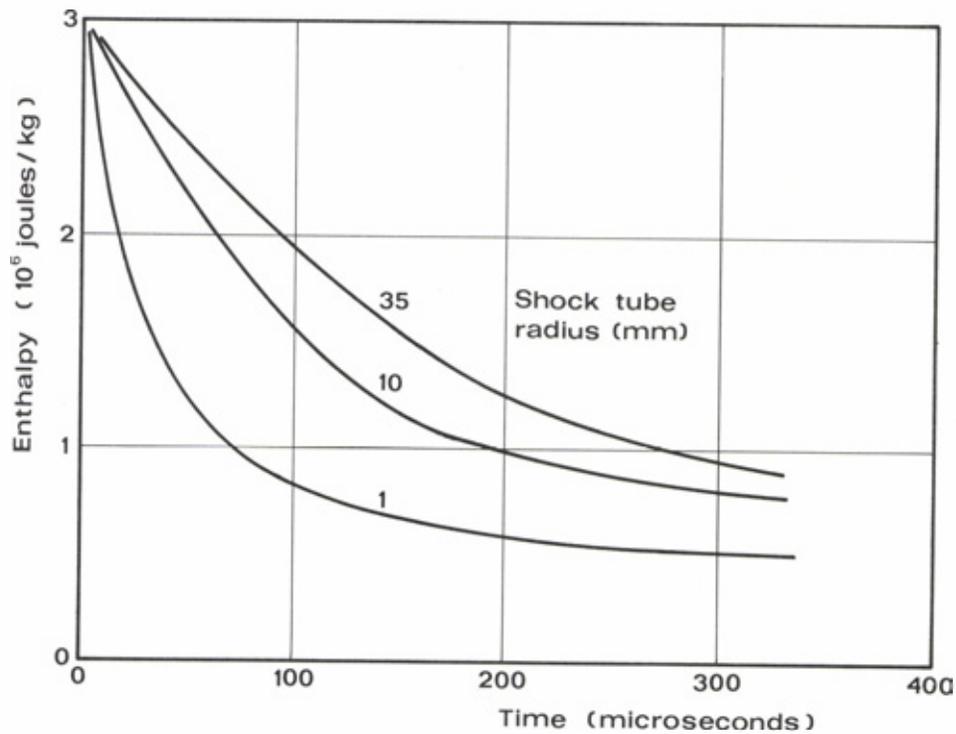


Fig. 9 Radiation Loss from Argon in Reflected Shock Region

Initial Temperature 20'000 °K
 Initial Electron Density $2.5 \cdot 10^{25}$ electrons/ m^3

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These results have been confirmed by experiments in the facility described above, and in a smaller shock tunnel. In both cases, maximum test section velocities obtained were approximately 7 km/sec, and were only 70% of what would have been achieved without radiation losses. An investigation of radiative loss processes in air indicates that a similar barrier to further velocity increases with reflected shock tunnels will occur at energies corresponding to velocities between 10 and 15 km/sec.

7. Non Reflected Operation - The Prior Steady Flow Technique

Although there is a barrier to what may be achieved in reflected shock operation, this does not apply to the basic facility itself. Indeed, the energies which have been obtained in the shock tubes at the Australian National University are such that, if the gas could be successfully expanded to hypersonic conditions velocities of 25 km/sec might be achieved.

This leads to consideration of non-reflected shock tunnel operation. In this case the shock tube is again fitted with a nozzle but, instead of expanding the gas from the stagnant region formed after shock reflection, the nozzle accepts the supersonic flow of gas between the shock wave and the interface, and expands it directly to hypersonic flow. This is shown in Fig. 10. Past research on this method (9) has established that it can be used to produce a satisfactory uniform flow. However, it has the disadvantage that the time between the passage of the shock wave and the interface past any point in the shock tube is very short and, because the nozzle flow can only be maintained during this time, the test times in the hypersonic flow also are very short.

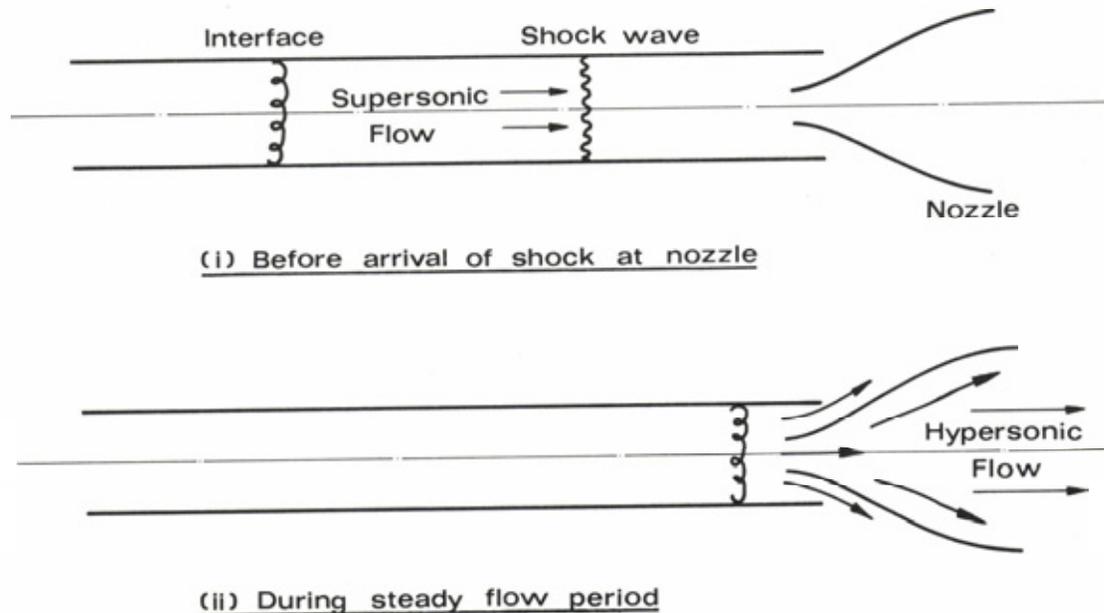


Fig. 10 Non-Reflected Shock Tunnel Operation

The shortest test times imply that special attention must be paid to the process of starting the flow in the hypersonic nozzle. The nozzle always will initially contain some "resident" gas and, in order to remove this resident gas before the main body of the test gas arrives, the momentum of the first layers of the test gas must be sufficient to accelerate all the resident gas to test gas speed. This implies that the density of the resident gas must be much less than the initial density of the test gas in the shock tube. In the past, this condition has been achieved by placing a thin diaphragm ahead of the nozzle, and evacuating the nozzle before a test. However this cannot be done with high performance shock tubes, as the time between passage of the shock wave and the interface is only of the order of 30 microseconds, and the diaphragm fragments cannot be removed from the flow in this time. Thus, an alternative technique has been developed (10), which is referred to as the "prior steady flow" technique. It involves establishing a steady flow of the test gas through the nozzle prior to arrival of the shock wave at the nozzle entrance. The steady flow in the hypersonic nozzle produces low densities over most of the nozzle length, and these low densities allow rapid starting of the nozzle flow.

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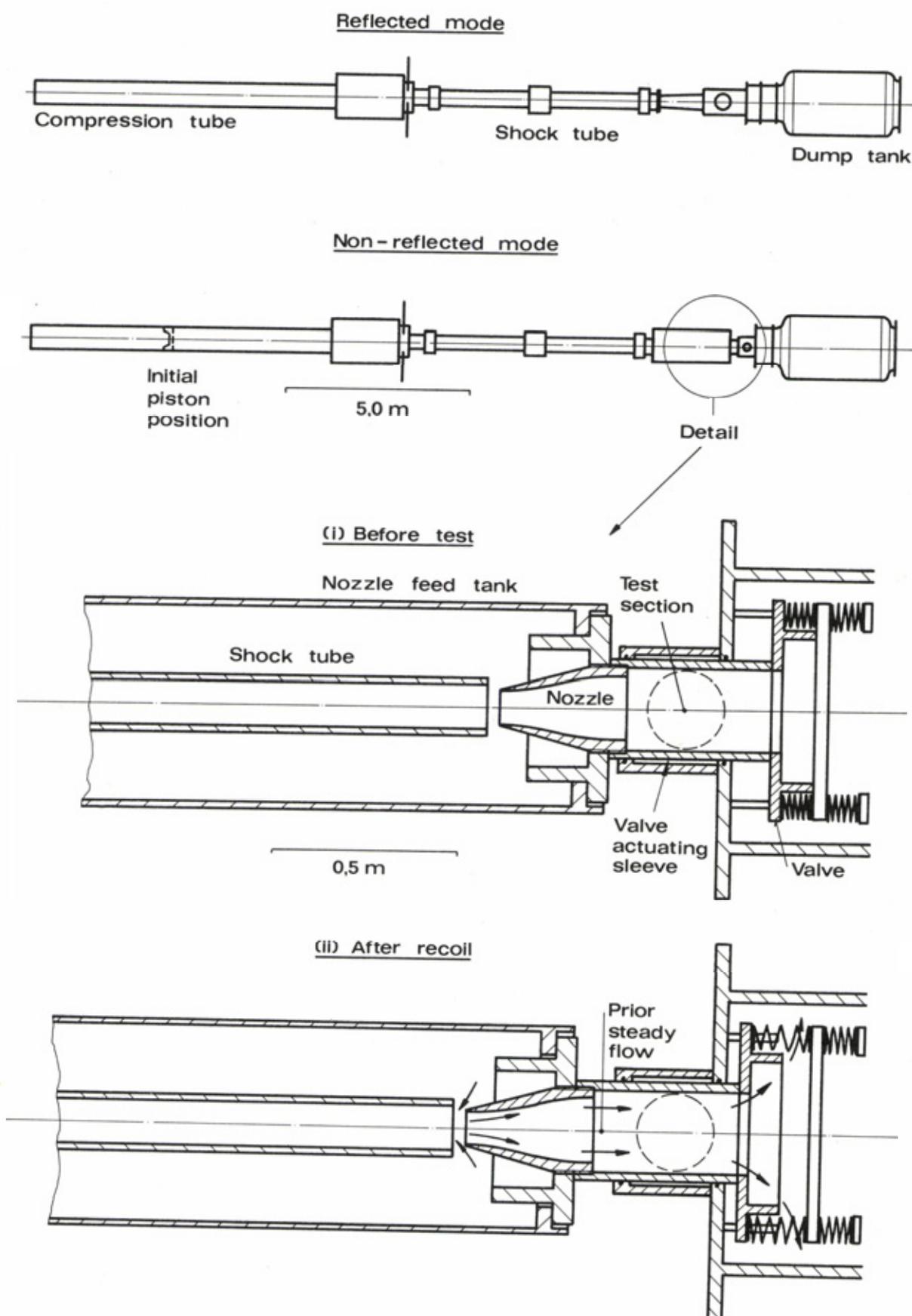


Fig. 11 Mechanism of Non-Reflected Shock Tunnel with Prior Steady Flow Operation

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A non-reflected shock tunnel, embodying the prior steady flow technique, is shown schematically in Fig. 11. This was designed as a modification to the existing shock tunnel (Fig. 7). As shown, the modification was effected by removing the nozzle and test section, which were used when operating in the reflected mode, and replacing them with a new test section, and an extension to the shock tube. The extension was surrounded by a nozzle feed tank, and the test section was vented to the dump tank via the valve assembly. The valve was spring loaded to open, and the valve actuating sleeve was rigidly connected to the shock tube. Prior to the test, the dump tank and test section assembly was moved towards the shock tube, allowing the valve actuating sleeve to force the valve closed, as in Fig. 11 (i). The dump tank assembly was then locked into position. The shock tunnel was fired by launching the piston. This caused the compression tube-shock tube assembly to recoil, and the valve actuating sleeve to retract, allowing the valve to open, as shown in Fig. 11 (ii). The dump tank had been previously evacuated, so opening of the valve provided a pressure differential across the nozzle, and the prior steady flow was initiated. The piston required 50 to 100 milliseconds to traverse the length of the compression tube, and this allowed the prior steady flow through the nozzle to become established. During this period, the nozzle feed tank acted as a reservoir for the prior steady flow, ensuring that there was no significant change in the initial shock tube pressure. When the piston reached the end of the compression tube, the shock tube main diaphragm was ruptured, and the shock tube flow was initiated.

This modification is, at the present time, still undergoing evaluation. However, time resolved interferograms of the flow in the test section have shown that the prior steady flow technique does indeed allow rapid starting of the flow in the nozzle, and that a steady flow can be maintained in the test section, for a period of at least 10 microseconds, at flow energies corresponding to velocities of 12 km/sec. Such test times are very short, and may be expected to reduce as velocities are increased. This means that it will not be possible to perform some experiments, particularly those involving phenomena which require some time to reach a steady state (e.g. experiments involving flow at the base of the vehicle). However, in the past, experimental data on heat transfer to the nose of blunt vehicles has been obtained in shock tubes test times less than 10 microseconds (11), and it seems clear that experiments of this nature will be possible.

8. Conclusion

Using a reflected shock tunnel, with an appropriate driving technique, it is possible to produce pulsed flows, in which the effects of chemical reactions in hypervelocity flows may be studied experimentally. These flows offer a test duration of the order of one millisecond, a time which is sufficient to allow investigation of a wide variety of flow configurations. However it is apparent that the radiation losses which are experienced in a reflected shock tunnel will limit the velocities which may be achieved to values between 10 and 15 km/sec. At present it appears that, in order to achieve higher velocities, it will be necessary to employ a non-reflected shock tunnel, embodying the prior steady flow technique for starting a hypersonic nozzle flow. Whilst the velocities which may be achieved by this method are limited only by the performance of the basic shock tube, the test times are very short, and it is possible that this may seriously limit the range of vehicle configuration which can be studied at speeds above 15 km/sec.

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